# Exam 4 Formula Sheet

- For a regular curve $\vec{r}(t)$,
  - $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$
  - $\vec{N}(t) = \dfrac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|}$
  - $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$
- For a regular curve $\vec{r}(t)$, the curvature is given by

$$\kappa(t) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}.$$

- **The Second Derivative Test for Functions of Two Variables**
  Left $f(x,y)$ be a twice differentiable function, and assume its second partial derivatives are continuous. Let $(a,b)$ be a critical point for $f$, and define the Hessian of $f$ at $(a,b)$ to be

$$D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2\,.$$

  - If $D(a,b) < 0$, then $(a,b)$ is a Saddle Point.
  - If $D(a,b) > 0$ and $f_{xx}(a,b) > 0$, then $(a,b)$ is a Local Minimum.
  - If $D(a,b) > 0$ and $f_{xx}(a,b) < 0$, then $(a,b)$ is a Local Maximum.
  - If $D(a,b) = 0$, the test is inconclusive.
- **Lagrange Multipliers**
  If a differentiable function $f$ has a local maximum or local minimum on a constraint curve of the form $g =$constant (where $g$ is a differentiable function), then there is a constant $\lambda$ so that

$$\nabla f = \lambda \nabla g$$

  at the local extremum (so long as $\nabla g$ is not zero at the point in question).
- **Polar Coordinates**
  - $x = r\cos\theta, \;\; y = r\sin\theta$
  - $r = \sqrt{x^2+y^2}, \;\; \tan\theta = \frac{y}{x}$
  - $dA = r\; dr d\theta$
- **Cylindrical Coordinates**
  - $x = r\cos\theta, \;\; y = r\sin\theta, \;\; z = z$
  - $r = \sqrt{x^2+y^2}, \;\; \tan\theta = \frac{y}{x}, \;\; z = z$
  - $dV = r\; dr d\theta dz$

- **Spherical Coordinates**
  - $x = \rho\cos\theta\sin\varphi, \quad y = \rho\sin\theta\sin\varphi, \quad z = \rho\cos\varphi$
  - $\rho = \sqrt{x^2+y^2+z^2}, \quad \tan\theta = \frac{y}{x}, \quad \varphi = \arccos(z/\rho)$
  - $dV = \rho^2 \sin\varphi \, d\rho d\theta d\varphi$
- **Change of Variables in Multiple Integrals**
  Suppose a transformation $T$ given by $x = g(u,v)$ and $y = h(u,v)$ is one-to-one (except perhaps on the boundary of $\mathcal{D}$), maps a region $\mathcal{D}$ in the $uv$-plane to a region $\mathcal{C}$ in the $xy$-plane, and $g$ and $h$ have continuous partial derivatives in a region containing $\mathcal{D}$. If $f$ is a continuous function over the region $\mathcal{C}$, then

  $$\iint_{\mathcal{C}} f(x,y)dx\,dy = \iint_{\mathcal{D}} f\left(g(u,v), h(u,v)\right) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv,$$

  where the Jacobian of the transformation is given by

  $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}.$$

  The analogous statement holds for triple integrals.